Coherent Motion in Excited Free Shear Flows

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Introduction

THE notion that coherent structures control the dynamics of all free shear layers was brought about by shadowgraph pictures taken in a plane turbulent mixing layer formed by two coflowing gases of different densities.[1] Coherent structures have been sought in plane mixing layers,[2-6] axisymmetric mixing layers,[7-10] plane jets,[11-13] wakes,[14,15] and, most recently, mixing layers generated by elliptical or other noncircular nozzles.[16,17]

As in most fast-evolving endeavors of research, controversy is not lacking because of the human ability to extrapolate and synthesize limited information based on scant data and to reshape it into a "universal" model or theory. We do not claim to be superhuman; therefore, this manuscript is bound to be biased and incomplete. We do not plan to present an historical compendium of all the data, models, and computations existing in the literature. We shall merely express our current point of view on the subject, accenting the possibilities of controlling the behavior of these flows by external manipulation of these large coherent structures. We shall start by describing qualitatively the various coherent structures and their dynamic significance.

Qualitative Descriptions of Coherent Structures

The plane mixing layer consists of an array of large eddies of concentrated vorticity extending many shear layer thicknesses in the spanwise direction (Fig. 1). The typical dimension of these vortices increases linearly with distance from the splitter plate, resulting in a linear growth of the shear layer.[1] These structures may disappear momentarily, depending presumably on the occurrence and the intensity of the uncontrolled extraneous disturbances. An amalgamation of adjacent structures, a process commonly referred to as "pairing," was observed at various degrees of regularity. Pairing was first discovered by Winant and Browand,[2] who injected a filament of dye into the flow. The dye rolled into lumps (marking the vortical fluid), which proceeded to pair with their neighbors, generating larger and more sparsely distributed structures. The occurrence of well-defined pairing is intermittent in both space and time.[1,3,4,18] Nevertheless, it is commonly believed that coalescence of vortices plays an important role in the growth rate of the mixing layer.

Secondary vortices oriented in the streamwise direction appear in the plane mixing layer some distance downstream of the splitter plate. The appearance of these vortices coincides with enhancement in the mixing process and is commonly referred to as mixing transition.[19,20] The streamwise vortices appear to ride on the primary eddies, retaining a preferred spanwise distribution.[20,21] The preferential location of the streamwise structures depends on the apparatus and can be altered by adding screens, turning vanes, etc. It is interesting to note that the longitudinal and the spanwise structures appear to coexist without visibly affecting one another.

The first concept of a large eddy in a plane wake originated with Townsend[22] and was reconstructed from two-point correlation measurements that were repeated at various angles to the flow. In the case of the wake, a double-barreled structure was postulated.[23] Recent observations[13,14,24] suggest that large coherent stuctures in fully developed plane jets and wakes are dominated by two rows of counter-rotating vortices that are staggered in the direction of streaming and span the flow (Fig. 2). These structures resemble the Kármán vortex street generated by a bluff body at low Reynolds numbers, but they do not retain the characteristic spacing and length scales[14] so prominent in the Kármán vortex street. The organized motion in fully developed turbulent plane jets is, in principle, similar to the coherent structures observed in a wake, but with an opposite sense of rotation.[11,12] The counter-rotating vortical structures appearing alternately on opposite sides of the plane of symmetry give the impression that both flows are flapping like a flag in the wind. The frequency and scale of the large structures change in such a manner as to satisfy the conditions of self-preservation.

The mixing layer generated by a jet emanating from an axisymmetric nozzle is initially similar in its behavior to a plane mixing layer, but as the flow evolves in the downstream direction, it becomes significantly more complex. First, the finite diameter of the nozzle introduces a length scale that is not present in the plane mixing layer and perturbations can be in-

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duced across the potential core. Second, the existence of many azimuthal modes greatly increases the range of possible interactions. The most frequently observed structures associated with an axisymmetric jet in the vicinity of the nozzle are vortex rings.[7,25,26] These structures are essentially axisymmetric and are spaced in a quasiperiodic fashion, as they are in the plane shear layer. One should be cautioned that most laboratory jets, by virtue of their design (i.e., a small nozzle emanating from a large settling chamber), may accentuate the axisymmetric mode of the large structures; this may not be as prevalent in industrial applications. Pairings or vortex amalgamations are observed in the shear layers of the axisymmetric jets, but the axisymmetry of the interactions is questionable.

All coherent structures have a dimension commensurate with the width of the shear flow and are therefore responsible for the large-scale transport of mass, heat, or momentum. They originate as flow instabilities of infinitesimal amplitude; but, once they become large and sufficiently energetic, they may interact with the mean flow and change its character. Coherent structures may also interact with one another; but, with the exception of "pairing," the nature of these interactions is only vaguely understood.

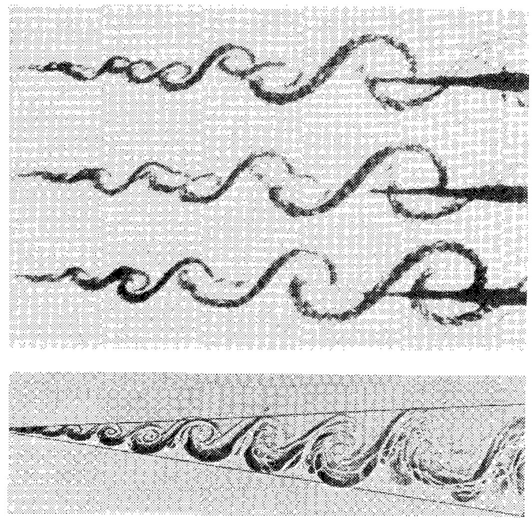


Fig. 1 Shadowgraphs of a mixing layer showing large coherent structures and a pairing interaction.[1,20]

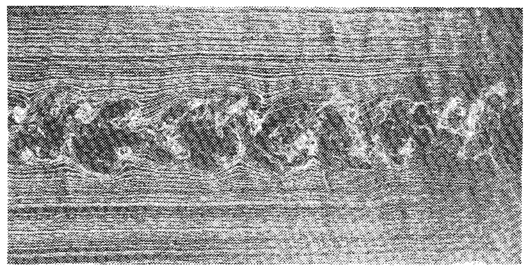


Fig. 2 Streaklines from a smoke wire showing the eddy structure in a plane turbulent wake.[15]

Linear Stability Theory Applied to Large Coherent Structures in Free Turbulent Shear Flows

Prior to the observations made in the turbulent mixing layer,[1,2] the statistical approaches to free shear flows consisted of various types of turbulence models. The new observations were significant because they enabled the flow dynamics to be described by a small number of modes. The coherent structures were first described as lumps of concentrated vorticity and their evolution with increasing time or distance was analyzed kinematically.[27,28] The small-perturbation instability approach for predicting the nature of coherent structures was not considered seriously, because it was commonly believed that the roll-up process (based on flow visualization) is highly nonlinear. More recent experiments[18,29,30] in artificially excited shear layers suggested that the data are well correlated with linear inviscid instability theory, in spite of the fact that the flows investigated were fully turbulent. Consequently, small-scale motions do not play an important role in the overall dynamics of flow development.[31]

On the basis of this premise, the spatial linear instability theory was extended to a slowly divergent, *prescribed mean flow*.[31,32] The calculations were compared in detail with an experiment in which two parallel streams of different velocity were allowed to mix.[31] Two-dimensional sinusoidal disturbances were introduced into the flow by means of a small oscillating flap mounted at the end of the splitter plate and the ensuing velocity field was measured with hot-wire anemometers. The distribution of the amplitudes of the ensuing oscillations in the streamwise and normal directions to the flow and the spanwise component of vorticity at two representative streamwise locations[32] are shown in Fig. 3. The symbols describe the experimental data and the solid lines show the theoretical results. Both sets of data were normalized by integrating the amplitude across the entire shear layer. The corresponding phase angles of the fundamental wave are superimposed on the amplitudes. The Reynolds stress, resulting from the instability wave, is plotted at the top of this figure, as is the shape of the mean velocity profile, which is assumed to be known within the realm of the linear stability approach. The agreement between the predicted and the observed *normalized distributions of amplitudes* is sufficiently encouraging to suggest that linear instability theory can be applied, at least as a first step in a series solution representing the role of large coherent structures in a turbulent mixing layer. A more thorough examination of the results reveals that:

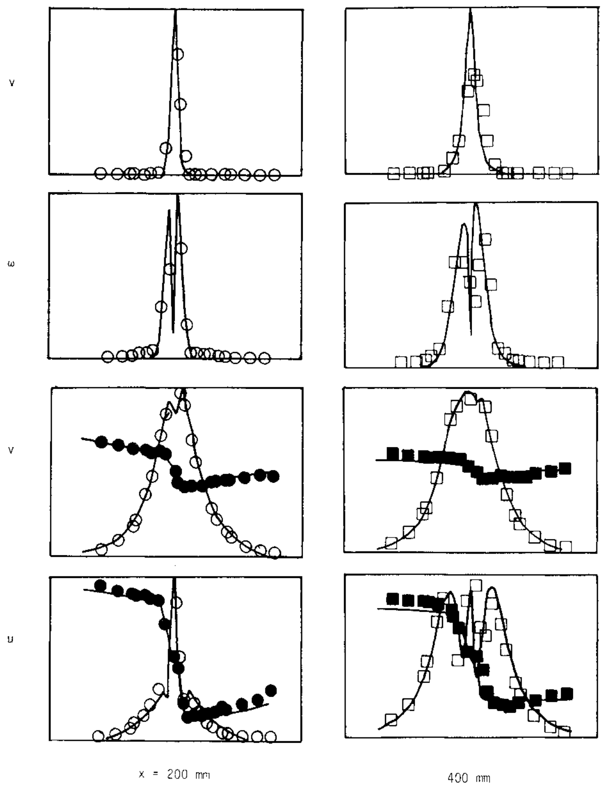


Fig. 3 Lateral distribution of phase-locked oscillations in a plane mixing layer at two distances from the splitter plate: —— comparison with theory.[32]

1) The amplitude distributions of the longitudinal fluctuations across the shear layer changes significantly with x, while the corresponding amplitudes of the normal fluctuations do

not. Near the splitter plate, where the shear layer is thin, the amplitudes of u are high at the center; but, as the flow becomes broader, the amplitudes near the center decrease, while the amplitudes at the peripheries of the mixing layer increase.

2) The phase velocity of the superimposed wave train is uniform across the shear layer provided the mean flow is assumed to be parallel. When the flow divergence is included, the theory predicts a variation in the phase speed with location.[31] In these computations, the phase advance is greater near the faster flowing stream than it is in the center or near the slower stream. The amplitude of the vorticity perturbation associated with this mode of instability has a minimum in the center of the shear layer (Fig. 3). If represented in the form of contours in the (y, t) or (y, x) plane,[33] it will indicate that the perturbation vorticity is concentrated in two lumps per wavelength, which are displaced relative to one another in both x and y (see also Fig. 9). Since their phase velocity is somewhat different, the vortex structures located near the high-speed stream will appear to ride on top of the slower moving vortex structures, so as to be rolled up and fused as the flow becomes neutrally stable to the excited wave train. Consequently, linear theory contains some aspects of the roll-up process. One may recall that broadband, space-time correlation measurements taken in the absence of external excitation indicated a similar pattern of behavior for large-scale structures.[34]

3) The measured and the calculated vorticity perturbation profiles have a minimum at the inflection point ($U''=0$) of the mean velocity profile. When the wave becomes neutrally stable, the vorticity perturbation changes from having two maxima to having a single maximum located at the inflection point.

4) The lateral distribution of $\langle u'v' \rangle_f$ is very narrow and is concentrated near the center of the shear layer (Fig. 3). The vanishing Reynolds stress elsewhere indicates that the u and v eigenfunctions are orthogonal; one should not infer that they are statistically independent.

5) The failure of the linear model to predict the overall amplification in the streamwise direction correctly[32] limits the range of its applicability.

In order to verify that the successful predictions provided by the instability approach are indeed universal and in order to check the limitations of the linear and inviscid approximations, the investigation was expanded to other important free shear flows: the small-deficit plane turbulent wake, plane turbulent jet, and axisymmetric jet.

Small-Deficit Plane Turbulent Wake*

The mean velocity profile in this flow has two inflection points and can therefore support two modes of plane instabilities, a sinuous mode and a varicose mode. The sinuous mode is the more unstable and the interactions between the two modes (linear superposition and a nonlinear resonance) pose some interesting questions currently under investigation.[15,24] Since the mean velocity gradients in this flow are rather weak, the ensuing amplification rates are small, enabling the investigators to follow the slow streamwise evolution of the large coherent structures. Furthermore, since the characteristic velocity scale is proportional to $x^{-1/2}$ and the characteristic width is proportional to $x^{1/2}$, the local Reynolds number does not vary with increasing distance from the wake-producing body. Consequently, one can easily assess the relative importance of viscosity (or the lack of it) on this flow.

Plane Turbulent Jet†

The linear modes of instability associated with this geometry are, in principle, the same as the instabilities existing in the wake. Nevertheless, the jet has numerous complications worth exploring. For example, the rates of amplification in this flow are much larger than in the wake and the local Reynolds number increases with increasing distance in the direction of streaming. The sinuous model of instability in the jet (as in the wake) has a much larger amplification rate than the corresponding varicose mode. A typical laboratory jet allows predominantly symmetrical disturbances to pass through the nozzle, giving a strong impetus to the initial evolution of the varicose mode, while the vortices shed alternately into the wake of a bluff body naturally introduce a strong sinuous disturbance into the flow. Although the frequencies of the predominant natural perturbations in the immediate vicinity of the nozzle or the body producing the wake seldom match the most unstable modes, the flow may become more unstable to these disturbances farther downstream as a result of changes in the width and the shape of the mean velocity profile.

Axisymmetric Jet‡

The most difficult test for the applicability of the linear instability approach to free turbulent shear flows is provided by the axisymmetric jet. The axisymmetric geometry admits the evolution of an infinite number of discrete azimuthal modes having a periodicity of 2π. In contradistinction of purely two-dimensional flows, in which the number of instability modes is limited to the number of inflection points in the mean velocity profile,[35] the azimuthal instability modes add a new degree of freedom to the axisymmetric configuration. The number of possible nonlinear interactions is therefore greatly increased because the Strouhal number based on the frequency of the perturbation is no longer the single independent variable of the problem.

During the initial evolution of the jet, the diameter of the nozzle adds an extra length scale to the analysis of this problem. Dimensional analysis indicates that the ratio of the radius of the jet R to the width of the shear layer θ has to be taken into account, in the same manner as the ratio of the boundary-layer thickness to the transverse radius of curvature is considered, whenever the boundary-layer approximation is applied to the flow around an axisymmetric body. The relative amplification rate of various azimuthal modes m also depends on the ratio θ/R. Cohen[10,36] proved that, whenever $\theta/R \ll 1$ and the dimensionless streamwise wavelength of the perturbation $\lambda_x/R \ll 2\pi/m$ (m is an integer), *all azimuthal modes will amplify at identical rates.* This condition is also equivalent to stating that $(\lambda_x/\lambda_\phi)^2 \ll 1$ (where λ_ϕ is the azimuthal wavelength). As the distance from the nozzle increases, the amplification rates of higher azimuthal modes ($m \geq 2$) decrease. The axisymmetric ($m=0$) and the helical ($m=1$) modes amplify at approximately identical rates over most of the core region; one anticipates, however,[37] that the helical mode will eventually dominate the flow farther downstream. The initial linear amplification of many azimuthal modes and the possible lengthy interaction between the axisymmetric and the helical modes, which may have approximately equal amplitudes, makes this geometry a unique test facility for determining the limitations of the linear theory.

Measurements were obtained in the transition region of a laboratory jet where azimuthal modes were excited by a phased array of speakers situated around the circumference of the nozzle.[10] The streamwise component of velocity was measured with a circumferential array of hot-wire probes. An example of the data obtained from this facility, which

*This investigation is being carried out at the University of Arizona with the cooperation of Prof. Francis H. Champagne.

†This investigation is being carried out by Prof. Chih-Ming Ho at the University of Southern California.

‡This investigation is being carried out at the University of Arizona.

represents (in our opinion) the usefulness of the linear theory, is herewith presented. For additional data obtained in the mixing layer, the plane wake, and the jet, the reader is referred to the relevant papers.

Axisymmetric excitation produced phase fronts that are constant in azimuth, while helical excitation produced phase fronts that advance one period in 360 deg of azimuth. Amplitude profiles were determined by Fourier transforming the phase-averaged measurements as:

$$F_{m,n}(r)=\frac{1}{2\pi T}\int_0^{2\pi}\int_0^{T}\langle u(\phi,t)\rangle \exp^{i[m\phi-(2\pi nt/T)]}\mathrm{d}t\mathrm{d}\phi \qquad (1)$$

where T is the period of excitation. Note that positive and negative values of m can be distinguished.

Figure 4 shows $F_{0,1}(r)$ and $F_{1,1}(r)$ for axisymmetric ($m=0$) and helical ($m=-1$) excitation, respectively. The corresponding stability eigenfunctions for both modes are shown for comparison. Since the $F_{m,n}(r)$ are complex functions, magnitudes and phase angles are shown. The magnitudes of the eigenfunctions are normalized so that the measured and theoretical profiles have equal areas. The theoretical phase angles have been offset to match the measured phase at the half-velocity point. The agreement between theory and measurement is quite good. The two sets of profiles, $m=0$ and 1, are virtually identical. This is in keeping with previous remarks about stability solutions being insensitive to low-order azimuthal modes when R/θ is large.

The measured growth of the disturbance amplitudes is shown in Fig. 5, where the integrated magnitudes are plotted vs streamwise distance for modes 0 and 1. The range of x/D is 0.3–0.7. Within this range, θ/R is relatively small, so the total amplifications of the two modes are about the same.

Linear stability theory based on parallel flow assumptions does a poor job of predicting the total amplification. The linear stability theory was extended to include weakly nonparallel terms for both the axisymmetric and nonaxisymmetric modes.[38,39] Figure 5 shows the results of the nonparallel analysis for mode 0 compared to the present measurements and compared to quasiparallel flow analysis. The quasiparallel calculation is based on the change in Strouhal number with downstream distance. The calculated quasiparallel amplification is about four times too large. This is reduced to a factor of about 1.4 when the nonparallel corrections are included. Although there is a great improvement, the remaining discrepancy between theory and measurement may be the result of energy extraction from the instability to other modes. For example, Stuart[40] has shown that first-order nonlinear corrections can lead to a finite-amplitude equilibrium. Another possibility is the coupling to the mean flow. These types of interactions are beyond the scope of the linearized theory.

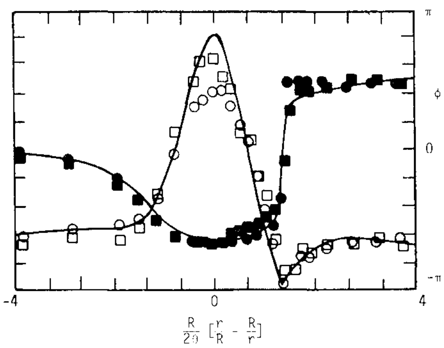


Fig. 4 Phase-locked amplitudes (open symbols) and phase distributions (solid symbols) in the shear layer of an axisymmetric jet at $x/D=0.4$: □ helical mode, ○ axisymmetric mode.[10]

Nonlinear Evolution of Instabilities in Free Shear Flows

By considering the linear instability of a slowly divergent mean motion, one tacitly acknowledges the nonlinearity of the process, because the divergence of the mean flow stems from the generation of Reynolds stresses and the latter represent second-order products of the perturbation. The depletion of the kinetic energy in the mean flow is *decoupled* from the energy gained by the perturbation by virtue of the linear approximation. In fact, the divergence of the mean flow stems from the loss of kinetic energy to the excited wave, which increases in amplitude. The exchange occurring between the mean motion and the unstable perturbation is but one example in the hierarchy of interchanges taking place among a variety of modes, wavelengths, or frequencies. In order that two waves will exchange energy efficiently, they have to satisfy kinematic resonance conditions.

Kelly[41] recognized the contribution of a periodic component superimposed on the mean flow in destabilizing a wave train having twice the wavelength of the original component. He refers to this process as a resonance condition contributing to a period doubling and perhaps reversing the usual cascade in turbulent flows. Kelly also decoupled the predominant fundamental wave from the growth of the subharmonic wave. In the nonlinear analysis developed by Cohen,[10] bilateral exchanges between various modes or between a predominant mode and the mean flow are considered. Therefore, the model can be easily expanded to a larger number of interactions, which can eventually be handled automatically on the computer.

A series solution incorporating a normal mode approximation to the inviscid equation of motion serves as a basis for this model. In two-dimensional flow, one uses the stream function to represent the solution, while the pressure is more conveniently used in conjunction with axisymmetric configurations. The series are expressed in terms of a small parameter ϵ, representing the amplitude of the oscillations, and have to be uniformly valid for the order considered. Since, in general, this type of representation gives rise to secular terms invalidating the descending order of the series, one has to introduce slowly varying length scales in the direction of streaming in order to eliminate the secular terms.

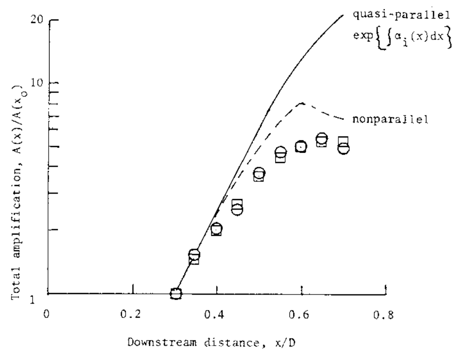


Fig. 5 The global amplification of $|u'|$ with streamwise distance in an axisymmetric jet: ○ $m=0$, □ $m=1$, —— comparison with theory.[10]

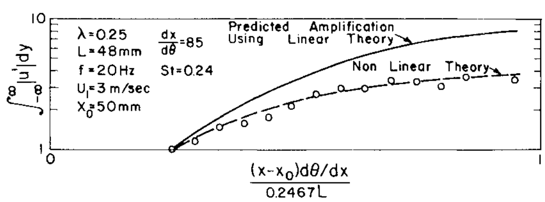


Fig. 6 Prediction of global amplification with streamwise distance in the plane mixing layer.

For example, when the amplitude of a wave becomes sufficiently large, the term of order ϵ^2 contributes to the generation of a harmonic frequency and to the spreading of the mean velocity profile. The mean flow interacts again with the fundamental wave train in a manner described by the slightly nonparallel, linear analysis.[31,38] The only difference is that the Reynolds stresses associated with the spreading originate from coherent, rather than random, motion and are therefore coupled to the amplification of the perturbation. By contrast, the linear analysis is based on a prescribed mean flow that is unaffected by the growth of the perturbation.

Applying this approach to one of the data sets given in Ref. 31 resulted in the improvement shown in Fig. 6. Moreover, the rate of spread of the mixing layer and the changes occurring in the mean velocity profile (Fig. 7) were accurately predicted. The theory also predicts the conditions for resonance and the nonlinear generation of a subharmonic frequency. In an axisymmetric mixing layer, resonant interactions between various azimuthal modes can generate new waves and can distort the shape of the jet through azimuthal modulation of the mean velocity profile.[10] Because low-frequency perturbations in a shear layer are dispersive, resonant interactions are limited to waves having frequencies in the range extending from the most amplified to the neutral point.[25] When the nondispersive azimuthal modes m_1 and m_2 interact (where m_1 represents the azimuthal mode number of the fundamental frequency and m_2 the mode number of the subharmonic), the resonance conditions will generate a subharmonic frequency that will have a mode number, $m_3 = \pm(m_1 - m_2)$. Thus, when one excites the jet with a helical mode rotating in the clockwise direction $m_1 = -1$ together with an axisymmetric subharmonic perturbation $m_2 = 0$, one can generate a subharmonic frequency that has a mode number $m_3 = 1$ rotating in the opposite direction (counterclockwise). An example of such an interaction is shown in Table 1, where the numbers refer to measured amplitudes. The top line in the table refers to the response of the jet to excitation at *either* mode $m = -1$ or 0. The second line shows the response to combined excitation at $m = -1$ and 0, resulting in the generation of mode $m = +1$. The experiment is described in Ref. 42.

The "Preferred Mode"

The predictive capability of stability theory to describe the dispersion relationship and amplitude profile of excited wave trains in the fully turbulent regime compels one to re-examine the concept of a preferred mode in axisymmetric jets. The concept was introduced by Crow and Champagne[7] in connection with their measurements in an excited jet. Their observations were inconsistent with the stability theories available at that time, so they concluded that their imposed disturbances had excited an unexplained instability of the entire jet column. They called the most unstable mode the "preferred mode." Since that time, spatial stability theory has advanced and most investigators feel that local Rayleigh instabilities play at least some role in jet turbulence.

The present concepts of a preferred mode[43,44] tend to describe a range of scales as Rayleigh modes that are confined to the transitional region. The preferred mode is characterized as a single scale that dominates the turbulent region. The frequency associated with the preferred mode is usually based on spectral measurements made near the end of the potential core and it is observed to scale approximately with the jet diameter and velocity.

The point of view that is consistent with stability theory is that the preferred mode is nothing more than a Rayleigh instability as observed at the end of the potential core. The observed scaling stems from the fact that high Reynolds number jets are kinematically similar and the passage frequency of a Rayleigh instability scales with the *local* momentum thickness rather than the initial momentum thickness. From this point of view, there is nothing more dynamically significant about scales observed at four diameters than scales observed at three or two diameters. A factor that might distinguish scales observed at four diameters from scales observed farther downstream is the decay of the centerline velocity. For jet noise considerations, for example, disturbances farther downstream are less energetic.

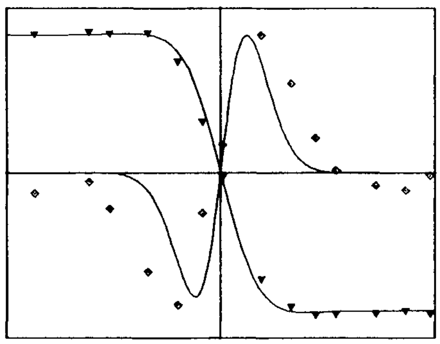
Gutmark and Ho[45] have compiled spectral measurements by various investigators of free jet disturbances near the end of the potential core. The reported Strouhal numbers $f\theta/U$ range 0.24-0.51. There are at least two explanations for this wide variation:

1) The confidence interval of the spectral estimates. The perturbations in free jets are aperiodic and so the spectral estimate will have a degree of uncertainty. For example, Petersen[25] obtained spectral estimates from the same data base that varied by a factor of two, depending on the processing technique used.

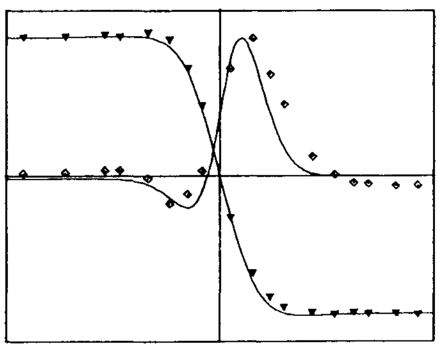
2) The effect of initial conditions. In all of these laboratory jets, the plenum chamber consists of a large volume and a small exit area. This combination of acoustic compliance and inertia results in acoustic modes that are axisymmetric (at low Mach numbers) and excite the natural jet instabilities. From the stability point of view, this not only influences which scales are amplified, but also affects the spreading rate of the mean flow. The geometry is also influenced by virtual origin variations at low Reynolds numbers.

Table 1 Example of subharmonic resonance, $m_1 = 0$ and $m_2 = -1$ (from Ref. 10)

	$f_2\theta/U_c = 0.04$			$f_1\theta/U_c = 0.08$		
Excitation	$m=0$	$m=1$	$m=-1$	$m=0$	$m=1$	$m=-1$
Individual	1.6	2.2	43.9	40.8	8.9	8.4
Combined	4.6	33.7	39.8	38.5	3.5	6.0



R = 0.25



R = 0.667

Fig. 7 Effect of excitation on the shape of the mean profile: — comparison with theory.[10]

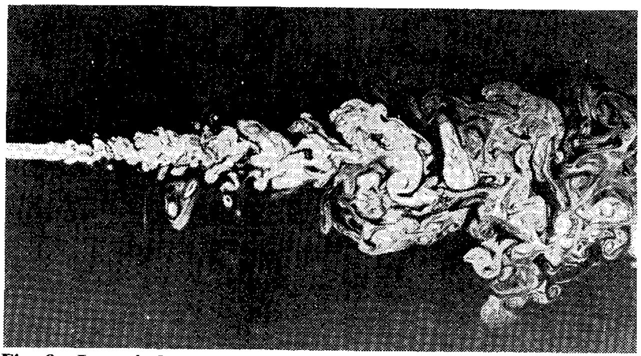


Fig. 8 Laser-induced fluorescence visualization of an axisymmetric jet.[47]

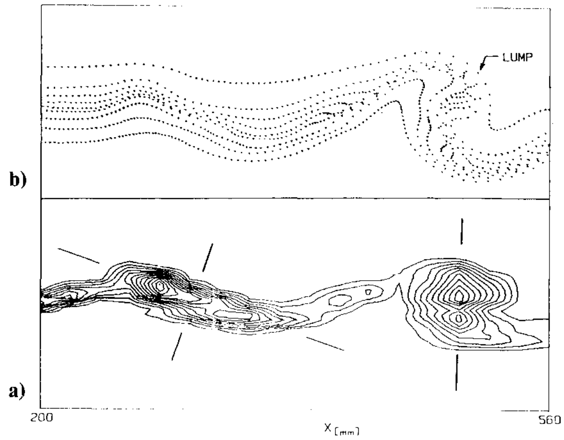


Fig. 9 Phase-locked streaklines and isodines measured in a plane excited mixing layer.[32]

Stability theory for parallel flow predicts that the most amplified wave train will be the wave with the maximum growth rate. When the divergence of the mean flow is included, the most amplified wave train differs from the wave with the maximum *local* growth rate. Crighton and Gaster[38] considered the stability of a tanh velocity profile and retained weakly nonparallel terms in their linearized equations. They concluded that the wave train with maximum total amplification at $x/D=4$ would have a Strouhal number near 0.38. This is certainly within the range of experimental observations.

In their model jet, the spreading rate $d\theta/dx$ was assumed to be 0.030 and the ratio of jet radius to momentum thickness R/θ was assumed to be 3.6 at $x/D=4.0$. Therefore, the Strouhal number, $fD/U=0.38$ corresponds to $2\pi f\theta/U=0.33$. Drubka[9] measured spectral peaks at the location $(x/D, r/D)=(4.0, 0)$ in a free jet. One may combine Drubka's spectral data and his momentum thickness data (Figs. 37 and 70 of Ref. 9) to compute a Strouhal number $2\pi f\theta/U$ in the range 0.32–0.39.

Kibens[44] observed deviations from Strouhal scaling at low speeds. He measured spectral peaks at $(x/D, r/D)=(3, 0)$ in a weakly excited jet and found that when the initial momentum thickness θ_0/R was greater than 1%, the frequency scales as $U^{1.6}$. Ho and Hsiao[46] observed essentially the same phenomenon in a planar jet. This is close to the scaling one would anticipate in a laminar jet. Namely, $f\theta/U=$ const and $\theta/D\propto R_D^{-1/2}$ so that $f\propto U^{1.5}$. In the case of Ho and Hsiao's measurements at $x/H=2$ (H is the nozzle width), this scaling seems to be associated with transition (Fig. 11 of Ref. 46, $U<65$ ft/s).

Another aspect of this question is the emergence of sinuous motion in the far jet. The original investigation by Crow and Champagne[7] was motivated in part by flow visualization experiments. The dominant features in their visualizations are the large axisymmetric structures visible near the end of the potential core. Figure 8 (from Dimotakis et al.[47]) is a laser-induced fluorescence visualization of an axisymmetric water jet. One might conclude that the dominant features in this visualization are the large sinuous structures in the free jet region. The length scale is of the order of 10 diameters and the Strouhal number based on diameter would be an order of magnitude less than 0.3.

It is not surprising that the most prominent features in the Crow and Champagne visualizations were axisymmetric. The acoustic properties of low Mach number plenum chambers are such that the natural excitation tends to be axisymmetric. However, the jet stability characteristics are such that axisymmetric disturbances are confined to the potential core region. Close to the jet exit, low-order azimuthal modes are all amplified at the same rate. However, as θ/R increases beyond 0.2, helical waves grow fastest.[36,48] Downstream of the potential core in the self-similar region, all axisymmetric waves are damped.[49] Although the dynamics of the jet beyond the end of the potential core has not been investigated, one would expect helical instabilities to be a dominant influence in this region. This idea is presently being investigated.

Vortex Pairing in Turbulent Mixing Layers

We have demonstrated that slightly nonlinear wave theory can predict many features associated with large coherent structures in the most basic free shear flows. The discussion would not be complete, however, if we did not try to reconcile our observations with the prevailing thoughts on the subject. Discrete vortex models are used to represent the shear layer, and vortex interactions are reported to be responsible for the lateral momentum transfer in turbulent mixing layers. Winant and Browand[2] have shown that an interaction takes place whereby two adjacent vortices merge to form a single structure. The characteristic width of the flow doubles because the circulation in a single vortex and the distance between adjacent vortices is approximately doubled as a result of the amalgamation. A continuous repetition of the pairing process controls, in their view, the growth of the mixing layer. Dimotakis and Brown[50] expanded on this idea by suggesting that the upstream influence of all large-scale vortices located at a given time on the axis of the flow is felt equally at the location at which the mixing process is initiated (i.e., at the trailing edge of the splitter plate). The idea stems from the application of the Biot-Savart law to a row of vortices whose circulation increases linearly with distance. Many models attempting to explain the spreading rate of the mixing layer are based on the global feedback effect stemming from the pairing of vortices and many experimental observations are attributed to this process.[6,9,30,51] The question arises whether the discrete vortex model can be made compatible with the nonlinear wave-instability approach.

Observations made in the highly excited mixing layer indicate that the rapid growth of the layer is associated with the amplitude of the excited disturbances. The mixing layer stops growing when the width θ of the shear layer is large enough so that the excited wave becomes neutrally stable.[30,52] If the amplitude of the excited wave is much higher than any other disturbance occurring naturally in the flow, the mixing layer will cease to grow until the other disturbances become comparable in amplitude to that of the excited wave.

The total vorticity in region I (i.e., the region in which the mixing layer spreads rapidly) is distributed in *two cores* that are displaced both laterally and longitudinally *per single wavelength* corresponding to the excitation frequency. The longitudinal distance between the cores diminishes as a result of the flow divergence and the two become displaced in y only when the shear layer becomes neutral to the disturbance frequency. (See Fig. 9.) This redistribution of vorticity is predicted by linear theory applied to a divergent mean flow,[33] but the divergence of the mean flow is implicitly a nonlinear process linked to the production of Reynolds stresses resulting from the finite amplitude of the wave. The authors believe that the bulk of experimental evidence suggesting that vortex amalgamation has a special dynamical significance not explainable by wave theory (including nonlinear wave interactions) stems from a particular method of visualization in which the dye or smoke is injected into the flow. Based on dye observations, Winant and Browand[2] proposed that pairing is caused by an instability of a discrete vortex array initiated by irregularities in the flow. It is a slow process in which two identifiable structures rotate around one another. A distortion takes place as a result of the interaction, with a concomitant diffusion at the adjoining interface. It should be remembered that visualization using tagged particles represents streaklines, which depend on the time and location of tagging relative to the time and location of the observations; therefore, it has to be treated with caution. An instantaneous method of visualization may be attained by using shadowgraphs sensitive to gradients in the index of refraction.[53] A sequence of shadowgraphs (courtesy of Prof. A. Roshko) exhibiting a pairing process was shown in Fig. 1. The dark lines in this figure represent the interface between the mixed, vortical fluid and the unmixed irrotational flow. The pattern resembles more closely the distribution of isodines shown in Fig. 9 and does not show the rotation of one lump of fluid around another. Although vortex amalgamation was not detected in Fig. 1d, the rate of spread of the shear layer did not seem to diminish. From shadowgraphs similar to those shown in Fig. 1, Hernan and Jimenez[54] concluded that most of the entrainment by the vortical fluid in the mixing layer occurs between pairings rather than during pairings.

Additional evidence that the observations of Weisbrot and Wygnanski[32] are consistent with the observations of amalgamation by Brown and Roshko[1] is provided by calculating (albeit approximately) the Strouhal number associated with the completion of pairing. The Strouhal number is evaluated from the average eddy spacing, the convection speed of the eddies, and the local width of the shear layer at the completion of pairing. The predominant Strouhal number calculated in this manner corresponds to the neutrally stable St for this flow.

Ho and Huang[6] excited the shear layer at a variety of frequencies, observed the evolution of large coherent structures, and measured some of the characteristic parameters in the flow. They concluded that whenever the shear layer was excited at a frequency f_f, which is equal or slightly lower than the natural frequency f_n at the initiation of mixing, vortex roll-up occurred at a locally neutral $St=f_f\theta/\bar{U}$ (where $\bar{U}$ is the average velocity of the two streams and θ the local momentum thickness) and the mixing layer did not diverge following the roll-up (see also Ref. 55, Fig. 3). This observation of Ho and Huang is consistent with the observations presented in Fig. 9.

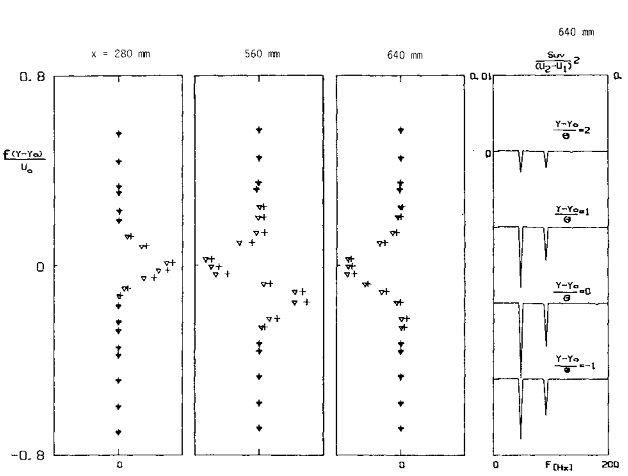


Fig. 10 Lateral distribution of Reynolds stress prior to, during, and after rollup, and the corresponding distribution of cospectra[32] showing only the excitation frequency and its first harmonic.

It is unfortunate that the phase-locked vorticity distribution leading to the roll-up was not measured by Ho and Huang.

Browand and Weidman[3] mapped the spanwise component of vorticity during a pairing process by using conditionally sampled data in an unexcited mixing layer. The contours presented in Figs. 6a and 6b of Ref. 3 are in good qualitative agreement with the contours plotted in Fig. 9 (provided one keeps in mind that the location of the high-velocity stream in the two sets of figures is reversed).

If one views pairing as a reorganization of vorticity within the shear layer with a concomitant roll-up of an undulating vortex sheet into a discrete lump (as shown in Fig. 9 at $x\sim500$ mm), the following corollaries ensue:

1) Rapid divergence of the mixing layer[55] and the generation of significant Reynolds stresses occur during the intermediate stages of pairing.[3] The rate of divergence is generally reduced with the completion of vortex coalescence. In the externally excited mixing layer, the intermediate stages of pairing coincide with the region in which the imposed perturbations amplify rapidly. The linear neutral stability point coincides with the location at which the mixing layer thickness reaches a plateau and where the pairing process is complete. In fact, the lateral distribution of Reynolds stress is partly negative following the completion of pairing and has a typical "S" shape (Fig. 10, $x=560$ mm). The slightly *nonlinear* inviscid stability theory applied to the linear neutrally stable wave superposed on a tanh profile predicts that the lateral distribution of Reynolds stress will be of the form $\overline{uv}\propto\mathrm{sech}^2(y)\tanh(y)$, which has the characteristic S shape. This is so because even the smallest nonlinearity produces a complex amplitude function in the normal mode decomposition $A(x)\phi(y)\exp[i(\alpha x-\beta t)]$, resulting in the generation of Reynolds stress when ϕ becomes real. The Reynolds stress predicted by the linear model vanishes identically at the neutral point. The extension of the theory to damped modes[56] enabled Cohen[10] to predict the total reversal in the sign of the Reynolds stress.

2) The redistribution of vorticity does not entail the generation of a subharmonic frequency because the linear stability theory does not require the existence of two frequencies in order to redistribute the vorticity as shown in Fig. 9. The fact that the subharmonic frequency is not produced by the amalgamation of vorticity can be demonstrated only in a highly excited shear layer (Fig. 10) in which all other perturbations are of small amplitude in comparison with the excited stability wave. If the prevailing frequency in this flow has an amplitude comparable to the background spectrum, the nature of the instability is such that whenever the flow becomes neutrally stable to the prevailing frequency, the subharmonic frequency is very strongly amplified and dominates the flow some distance downstream. Since shadowgraphs (Fig. 1) delineate the boundaries of the vortical fluid, the disappearance of the boundary between adjacent structures in Figs. 1a and 1b does not necessarily imply a change in frequency.

If this view of pairing is to be adopted, then vortex amalgamation is simply another manifestation of flow divergence; it is not the cause of the divergence, as is still commonly described in the literature. The divergence of the flow can be correlated with nonlinear effects of wave instability. It occurs whenever the amplitude of the prevailing oscillation is sufficiently high, whereby its cross product contributes to the mean motion and to the generation of a harmonic frequency. The rate of amplification of the harmonic frequency is initially high, because this wave derives its energy by the linear process from the mean motion and by the nonlinear process from the prevailing wave. The linear amplification stops whenever the shear layer becomes too broad for the wavelength considered, but the harmonic frequency can still gain some energy by the nonlinear process. When Ho and Huang[6] excited their shear layer at a frequency f_f, which was less than half of the natural frequency f_n at the initiation of mixing, the harmonic wave

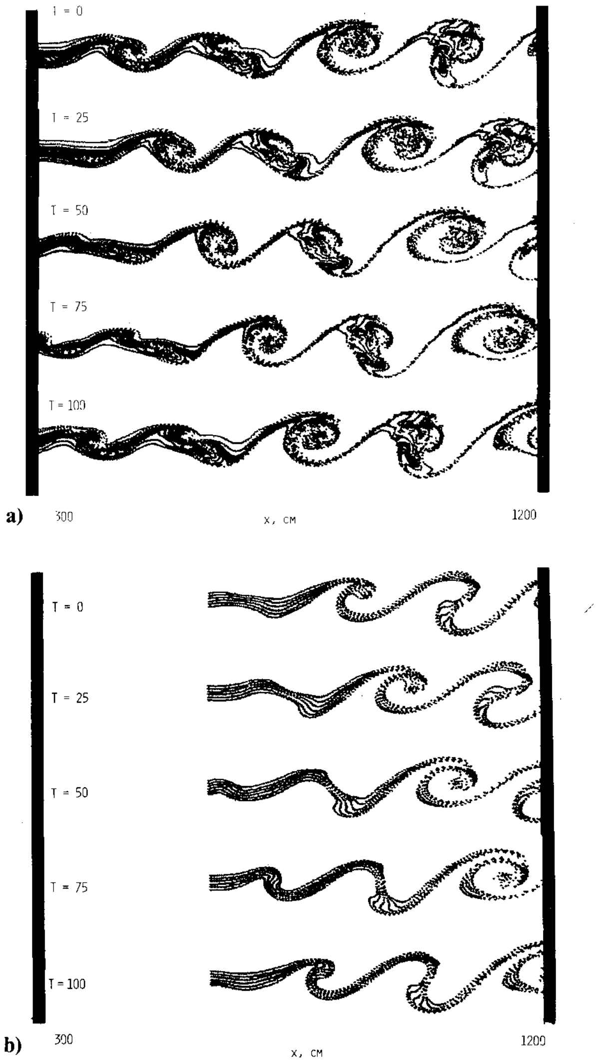


Fig. 11 Streaklines calculated from measured velocities in a plane mixing layer excited at two frequencies an octave apart (streaklines are phase locked to the lower frequency).

$f2_f \simeq f_n$ (which must have had a high amplitude to begin with) became the predominant wave in the flow. When the mean motion became sufficiently broad, this wave became neutrally stable, rolling up the particles of fluid into lumps. The excited wave retained its rate of amplification until, at some location farther downstream, it also became neutrally stable. Since the tagged fluid particles were already rolled into discrete lumps by the harmonic wave, the roll-up of particles associated with the excitation frequency appeared as vortex amalgamation (see Fig. 19 of Ref. 55). The redistribution of vorticity at this location might have been dominated by the excitation frequency, as was discussed previously. The more complex visual appearances of merging vortices may stem from the fact that Ho and Huang[6] observed streaklines generated at the trailing edge of the splitter plate. If, for example, the dye in their experiment had been injected farther downstream at the location at which the amplitude of the harmonic frequency was approximately equal to the amplitude of the excited wave, the streakline pattern might have been different. It was pointed out by Cimbala[14] that streaklines generated in the wake of a circular cylinder may show patterns which do not exist locally in the flow. A similar phenomenon was observed when a mixing layer was excited simultaneously by two wave trains having a frequency ratio of 2 (i.e., $f_f = 2f_s$). The streaklines in this experiment were calculated from measured velocities that were phase locked to the subharmonic frequency. When the streaklines originated near the trailing edge of the splitter plate, the pairing of streaklines—and presumably of accumulated material (dye or smoke) that had earlier (i.e., at a smaller value of x) rolled up into lumps at a wavelength associated with the high frequency—was observed. When the origin of the streaklines was shifted downstream, the same flowfield (i.e., identical experimental data) produced a streakline pattern that was dominated by the roll-up at the lower frequency f_s and showed no sign of pairing. See Fig. 11.

The suggestion that flow visualization without the simultaneous knowledge of the vorticity distribution might at times be confusing was explored by computer simulation. The streaklines drawn in Fig. 12 show a simulated "pairing" interaction. A neutrally stable wave, when superimposed on a "tanh" velocity profile, produced a streakline pattern resembling a parallel row of rolled vortical regions (Fig. 12a; see also Ref. 57). The streaklines in Fig. 12b were calculated on the basis of the linear stability theory for the rapidly amplified subharmonic frequency. Although the initial amplitude of the subharmonic wave is only one-tenth the amplitude of the neutrally stable wave, the lateral excursions of the particles some two wavelengths downstream of the origin are larger than those of the corresponding neutral wave. Superposing the two waves and recomputing the streaklines produces a visual appearance of a "pairing" interaction shown in Fig. 12c. The coalescence begins at a distance from the origin corresponding to approximately two wavelengths of the subharmonic frequency and is completed at a distance of five wavelengths from the origin. The corresponding isodines plotted in Fig. 13 do not indicate a redistribution of vorticity during the period in which the tagged particles (i.e., streakline patterns) amalgamated. Since the subharmonic wave is continuously amplified during amalgamation, the resulting vorticity pattern is dominated by the subharmonic frequency. The isodines plotted in Fig. 13b (which correspond to the flow pattern plotted in Fig. 12b) do not differ appreciably from the isodines calculated during the amalgamation process (Fig. 12c). *The lateral dimensions in Fig. 13 are stretched* in order to shown the details of the phenomenon more clearly. All of the contours plotted in Fig. 13 have identical numerical values. The maximum vorticity concentration in Figs. 13b and 13c increases by two orders of magnitude over the distance plotted.

This exercise simulates the linear amplification of the subharmonic frequency in region II, which does not produce a strong coherent cospectrum at this frequency.[32] The streakline pattern in Fig. 11a agrees very well with the phase-locked smoke filaments observed visually in the excited shear layer[32] and with the streaklines calculated from the phase-locked velocities (Fig. 9).

One may easily simulate a variety of amalgamations by superposing waves of different frequencies, amplitudes, phase velocities, and amplification rates, as well as initial phase angles. The patterns produced resemble all of the modes discussed by Ho and Huang[6] and even the "collective interaction" process, or a tearing process, is well simulated (Fig. 14). The choice of the initial phase angle is not important whenever the phase velocity of each of the two waves participating in the process is different (i.e., the waves are dispersive) and the distance covered by these computations is large. Calculations based on the temporal evolution of the flow[28,58] overemphasize the importance of the initial phase angle because temporal waves are nondispersive.

The evidence presented suggests that amalgamation of tagged fluid particles does not necessarily coincide with the redistribution of vorticity and may therefore be of no dynamical significance. Furthermore, one can simulate "pairing," "tripling," and "collective interaction" as the kinematic distortion of a spatially growing wave train superimposed on a mean shear. This calculation does not preclude the possibility that vortex pairing and vorticity redistribution may coincide in a divergent shear layer. The evidence accumulated thus far suggests that a significant fraction of the overall Reynolds stress is produced by a slightly

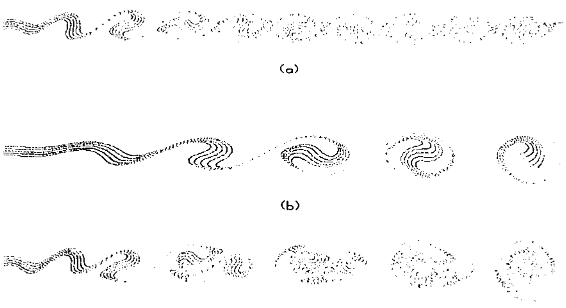


Fig. 12 Simulated streaklines exhibiting pairing.[32]

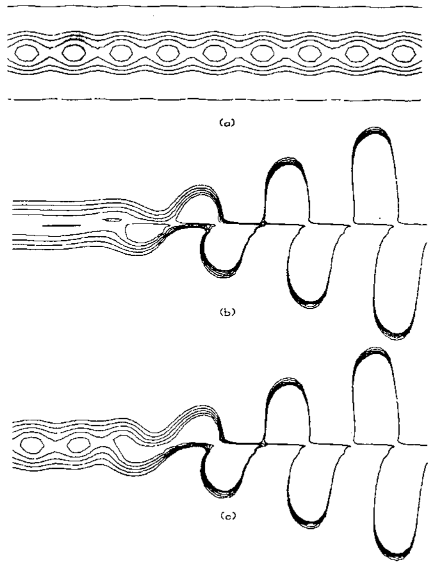


Fig. 13 Simulated isodines during pairing.[32]

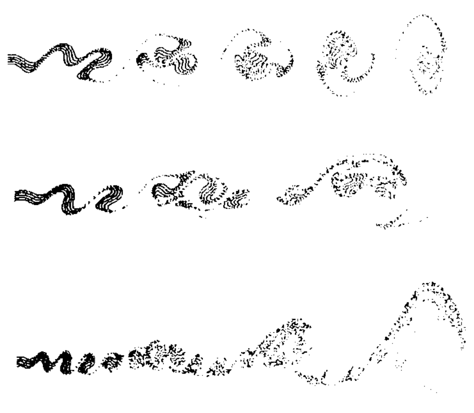


Fig. 14 Simulated pairing, tripling, and collective interaction.[32]

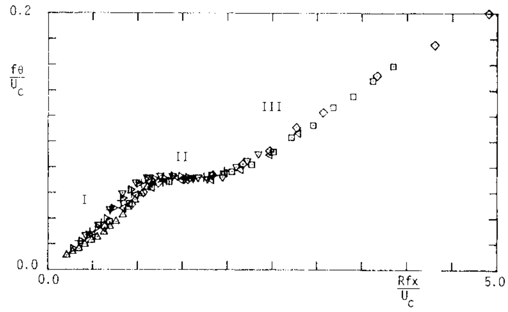


Fig. 15 Divergence of the plane shear layer with downstream distance. $U_1/U_2 = +0.6$, ◁ 0.5, □ 0.4, ◇ 0.3; $R = (U_1 - U_2) \div (U_1 + U_2 6)$; $U_c = (U_1 + U_2)/2$.[10]

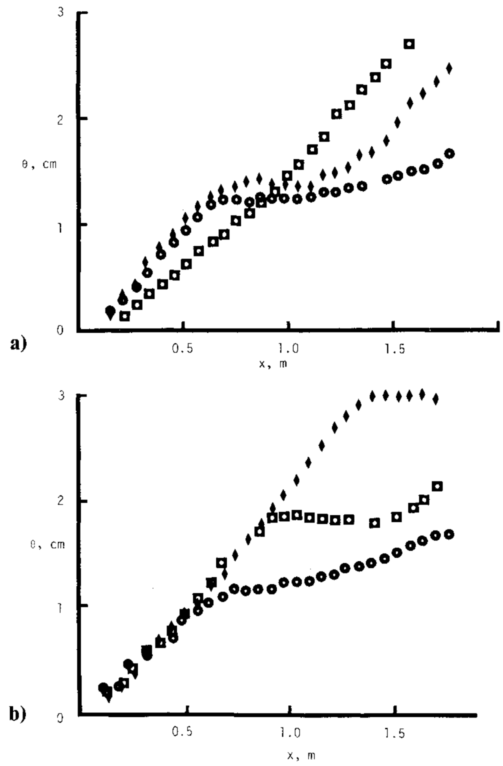


Fig. 16 Lateral growth of the mixing layer excited by two frequencies: a) small-amplitude subharmonic; b) subharmonic amplitude is doubled (○ fundamental, □ subharmonic, ◆ combined).[67]

nonlinear process which can be described by a vortex model or by the instability approach; *the two methods of description are not imcompatible.* The viability of a specific method will be determined by its ability to predict the behavior of the shear layer quantitatively.

Control of Free Turbulent Shear Layers: Experimental Results

The first significant alteration in the growth rate of a fully developed, turbulent, plane mixing layer at moderately high Reynolds numbers was made inadvertently.[59] A trip wire was placed on a splitter plate just upstream of the initiation of mixing, enhancing the rate of spread of the flow by approximately 30% relative to classical results published by Liepmann and Laufer.[60] Although the results were reproduced by Batt et al.,[61] they were regarded with skepticism since the universality of self-preserving turbulent flow was considered unquestionable, even for engineering applications. Experiments aimed at controlling the spreading rate of the mixing layer by simple passive devices such as trip wires were reported in the literature.[34,62,63] Tripping devices are not the most effective means of excitation; they can enhance the growth rate by 30% or retard it by approximately 15%, depending on the velocity ratio between the two streams, the diameter of the boundary layer on the splitter plate, the initial momentum thickness, and the uniformity of the freestream. Two-dimensional trip wires are more efficient in changing the rate of spread of the mixing layer than three-dimensional tripping devices; furthermore, putting trip wires on both sides of a splitter plate affected the initial momentum thickness of the shear layer, but not the rate of its growth.

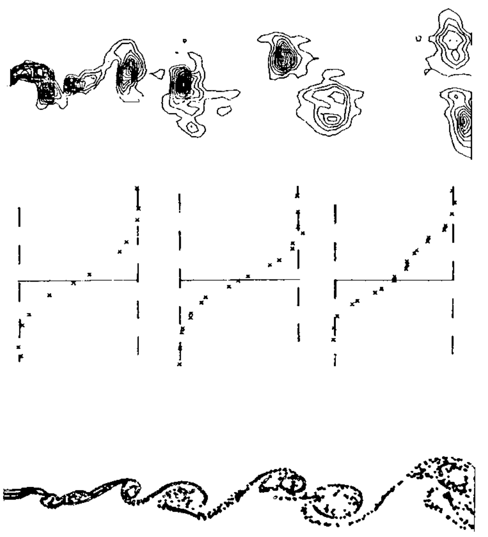


Fig. 17 Streaklines, isodines, and velocity profiles corresponding to Fig. 16b.[32]

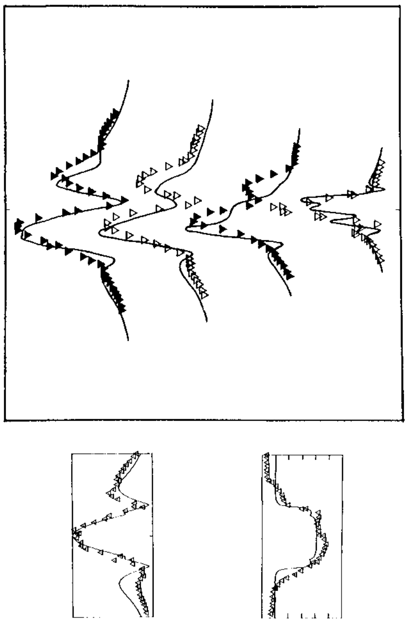


Fig. 18 Excited modes in a small-deficit wake: a) varicose mode: amplitude and phase; b) varicose and sinuous modes combined: evolution of amplitude profiles.[68]

Coherent, active, external excitation can be produced by many means, depending to some extent on the medium considered (i.e., whether it is gas or liquid). One may perturb the flow by using mechanical devices (oscillating flaps, vibrating ribbons, piezoelectric materials, etc.), hydrodynamic techniques (periodic suction or blowing), or acoustical excitation, thermal techniques (heating strips, electrical discharge) or by modulating the mean flow. The excitation can take the form of a wave train, wave packet, pulse, or combination thereof.

One can achieve some direct control over global features of the flow (such as entrainment rates) using these techniques. One can also achieve indirect control over molecular mixing to the extent that it is affected by the dynamics of the large scales. Such control has obvious technological significance for applications such as combustion, performance of ejectors, and high-lift devices.[64,65]

Oster et al.[4,18] presented a series of measurements in a turbulent mixing layer excited at relatively low frequencies. Their results indicate that the initial rate of spread of the flow can be significantly increased. The divergence of the flow with distance from the splitter plate depends on the velocity ratio $R=(U_2-U_1)/(U_2+U_1)$, the amplitude of the flap oscillation A_o, the frequency of the excitation f_f, and the mean velocity of the two streams $U_c=(U_1+U_2)/2$. Two-dimensional oscillations of fairly small amplitude tend to increase the rate of spreading of the flow until a neutrally stable Strouhal number (f_θ/U_c) is attained, whereby the rate of divergence will decrease.

At higher amplitudes of excitation, the mixing layer saturates, retaining a constant θ and yielding an approximate value of $f_\theta/U_c=0.075$. An additional increase in the amplitude of excitation may cause an overshoot in the maximum St attained, which is followed by contraction to a lower value of St. The amplitude of the excited wave in the saturated region may exceed by an order of magnitude the amplitudes of the background turbulence level. The saturated region extends usually over a distance[18,54] $1<R(fx/U_c)<2$. Therefore, the mixing layer was subdivided into three distinct subregions, marked by Roman numerals in Fig. 15. The saturation of the momentum thickness can be predicted using the nonlinear instability approach discussed. The method also predicts the changes in the shape of the mean velocity profile occurring as a result of the excitation (Fig. 7). These changes depend on St and R. The overall turbulent intensity was observed to increase in the excited shear layer,[18] but the gains in the background turbulence are not uniformly distributed among the three components of the velocity fluctuations. The intensity of the spanwise $\overline{w^2}$ fluctuations is suppressed by the periodic surging, suggesting that the flow becomes more two-dimensional. The intensity of the normal fluctuations $\overline{v^2}$ is greatly increased, but the lateral distribution of $\overline{v^2}$ is not significantly affected by the excitation. The intensity of the streamwise component $\overline{u^2}$ is increased by the excitation and its lateral distribution is significantly changed in region II, as was predicted by the instability approach.

An important discovery was made by Roberts[66] while investigating a chemically reacting mixing layer excited by a periodic perturbation. The amount of the product resulting from the chemical reaction was measured by absorption of laser light with the help of a self-scanning photodiode. The rate of chemical reaction was increased in region I at fairly large values of x, beyond which the flow ceased its simple "flopping" motion caused by the excitation. In region II, where *the shear layer stopped growing, chemical reaction ceased as well.* It is surprising that chemical reaction associated with molecular scales is so closely tied to entrainment generated by large coherent structures. It underlies, however, the importance of large-scale structures in combustion and chemical reactions.

So far we have discussed the effect of simple periodic excitation on the plane mixing layer. By exciting the flow simultaneously at two or more frequencies in the nonlinear regime, one can greatly extend the range over which the flow can be controlled.

The growth of a mixing layer excited simultaneously by two frequencies, f_f and $f_s=\frac{1}{2}f_f$, but differing in amplitude is plotted in Fig. 16.[67] In Fig. 16a, the amplitude of the subharmonic velocity perturbation is estimated to be 23% in comparison to the amplitude of the fundamental perturbation. The fundamental excitation by itself causes the shear layer to saturate in the usual manner at $f_1\theta/U_c=0.07$. The subharmonic perturbation, being rather small, amplifies in accordance with linear instability until it attains a sufficiently high amplitude to increase $d\theta/dx$ beyond its nominal unforced value. The mixing layer excited by the two frequencies combined is initially dominated by the fundamental frequency and saturates at approximately the same $f_f\theta/U_c$ as before. The value of Rf_fx/U_c is, however, different and the distance x during which the mixing layer is saturated is much shorter. In region III, however, the rate of spread of the shear layer excited by both frequen-

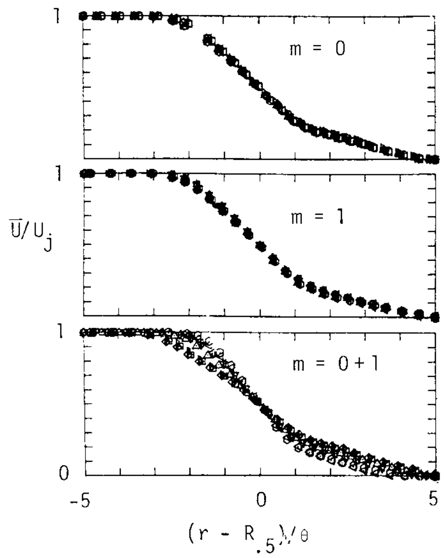


Fig. 19 Azimuthal similarity of mean velocity profiles in a flow produced by an axisymmetric nozzle ($x/D = 0.65$).[10]

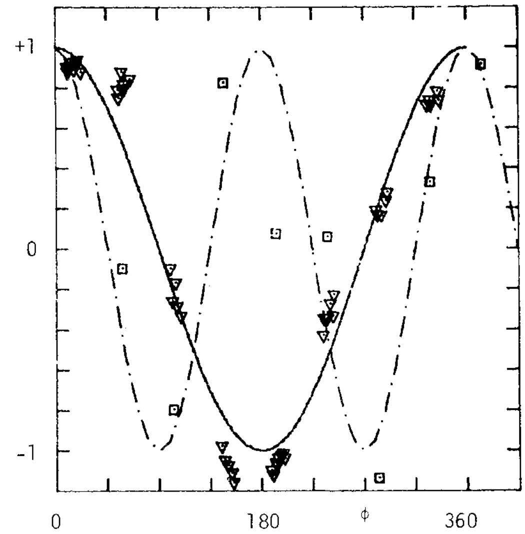


Fig. 20 Azimuthal modulation of the mean velocity profile produced by resonant interactions: ▽ modes 0 and 1, □ modes 0 and 2.[10]

cies is equal to $d\theta/dx$ of the shear layer excited by the subharmonic f_s. When the amplitude of the excitation was increased in such a manner that the relative amplitude of the subharmonic was approximately doubled, the shear layer behaved quite differently. When excited by the fundamental frequency or the subharmonic frequency alone, the shear layer saturated approximately as expected (the subharmonic excitation had a larger level of a harmonic distortion stemming from the mechanical flap arrangement). Under combined excitation, the shear layer retained its linear growth well in excess of the normal prediction based on the instability approach. In this case, the mixing layer rolls into discrete vortices by the excitation at the fundamental frequency and the latter are flung in the lateral direction by the high-amplitude subharmonic oscillation. The effect manifests itself in a velocity profile that has a hump at its center (Fig. 17), in streaklines that appear to pair (Fig. 17), yet with concomitant vorticity concentrations that are separating in the lateral direction. This type of flow might be dominated by nonlinearities, which are currently being investigated.

The exploration or exploitation of the primary instability of free shear by external means was referred to as "excitation." One may, however, "force" a shear flow by exciting the secondary modes of instability or by exciting the primary modes at such a high amplitude as to produce strong nonlinearities at the source of the excitation. The secondary (varicose) instability in the wake was forced into dominance by Champagne et al.[68] The varicose mode dominated the wake for a distance equivalent to 1400 momentum thicknesses (Fig. 18) and it never disappeared. When both modes were excited, the resulting turbulence level in the wake was asymmetric throughout the region of investigation. A second example of forcing is the blooming jet experiment of Reynolds[69]; in this case, the level of excitation is well beyond the saturation level of the initially predominant instabilities. The jet initially undulated around some mean position, distributing vortices produced by the forcing; but, when the broad mean flow was established by the interacting vortices, a helical instability mode might have established itself as a result of the azimuthal precision of the nozzle. This effect might have been responsible for the unprecedented rates of spread of this flow.

In the mixing layer produced by an axisymmetric jet, one has the added freedom of controlling the flow by generating resonant interactions between different azimuthal modes. The most dramatic resonant interaction occurs when two waves are excited at the same frequency but at different mode numbers.[42] In this case, the interaction produces a steady azimuthal variation of the mean flow.[10,70] Figure 19 shows a family of mean velocity profiles measured at 45 deg intervals in azimuth. The radial coordinates are each normalized by the local momentum thickness and centered at the half-velocity point. From Fig. 19, it is clear that the mean velocity profiles were axisymmetric when the jet was excited at either $m = 0$ or 1. However, when both modes were excited simultaneously at the same frequency, there were substantial deviations from axial symmetry (Fig. 19).

From the resonance conditions, a resonant interaction between two waves with the same frequencies and streamwise wave numbers produces *a third wave, steady in time,* with azimuthal mode number $m_3 = \pm(m_1 - m_2)$. Consequently, when $m_1 = 0$ and $m_2 = 1$, then $m_3 = \pm 1$ and the azimuthal modulation should be of the form $\cos(\phi)$. Figure 20 shows the measured modulation compared to $\cos(\phi)$. The quantity $\Delta u(\phi)$ is the change in mean velocity with and without excitation. Seven radial positions are included and the velocity increments are normalized by the maximum Δu for each radius. The modulation is essentially cosine, as expected.

Also plotted in Fig. 20 is the mean velocity modulation when the jet was excited at $m_1 = 0$ and $m_2 = 2$. The measurements show reasonable agreement with the expected $\cos(2\phi)$ modulation. The distortion of the mean profile may be of engineering significance because one should be able to distort the shape of a circular jet and increase or decease preferentially its ability to entrain surrounding fluid.

There recently has been a strong interest in the mixing layer generated by elliptic jets[16,17] because they appear to be capable of entraining surrounding fluid more efficiently than corresponding circular jets, particularly when the aspect ratio was moderately small ($AR = 2{:}1$). One of the interesting phenomena associated with these jets is a switchover between the major and minor axes of the mean flow occurring downstream of the nozzle. The number of switchovers observed and their preferred location depend on the geometry and flow conditions. It is felt that one can generate a quasielliptic jet by exciting a circular jet at modes 0 and 2. External excitation of jets may therefore be coupled with geometrical nozzle design in augmenting the mixing produced by complex nozzle shapes.§

§An investigation of the flow emerging from noncircular nozzles (e.g., triangular, square, etc.) is ongoing.[71] Initial observations suggest that a noncircular flowfield may indeed produce a combination of modes as predicted by Cohen and Wygnanski.[42]

Conclusion

In conclusion, we have explored the application of the inviscid instability approach to externally excited turbulent free shear flows at high Reynolds numbers. In this context, we found the special features attributed to "pairing" or to the "preferred mode" hard to comprehend and the concept of "feedback" in need of further substantiation in incompressible flow. Some of the concepts, if quantitatively defined, may perhaps be used as alternate descriptions of the phenomena observed and, in the absence of external excitation, both approaches need further substantiation. We have initiated the exploration and exploitation of a myriad of nonlinear effects among various modes of instability that open new vistas in the control of turbulent free shear flows.

Acknowledgments

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